Monte Carlo Simulation of Positronium Cooling for Bose-Einstein Condensation

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Positronium and its Bose-Einstein Condensation (BEC)

Positronium (Ps)

- The bound state of an electron (e-) and a positron (e+).
- The lightest and exotic atom.

Ps BEC

High critical temperature.
Our target: 14 K @ 10^{18}/cm^3

Scientific goal and application

- **Antimatter gravity measurement**
- **Gamma-ray laser**

$$n\lambda_D^3 = n\left(\frac{2\pi\hbar^2}{mk_B T_C}\right)^{3/2} = 2.612, \quad (1)$$

n : number density,
λ_D : de Broglie wave length,
$\hbar$: reduced Planck constant,
m : atom mass,
k_B : Boltzmann constant,
T_C : BEC critical temperature.

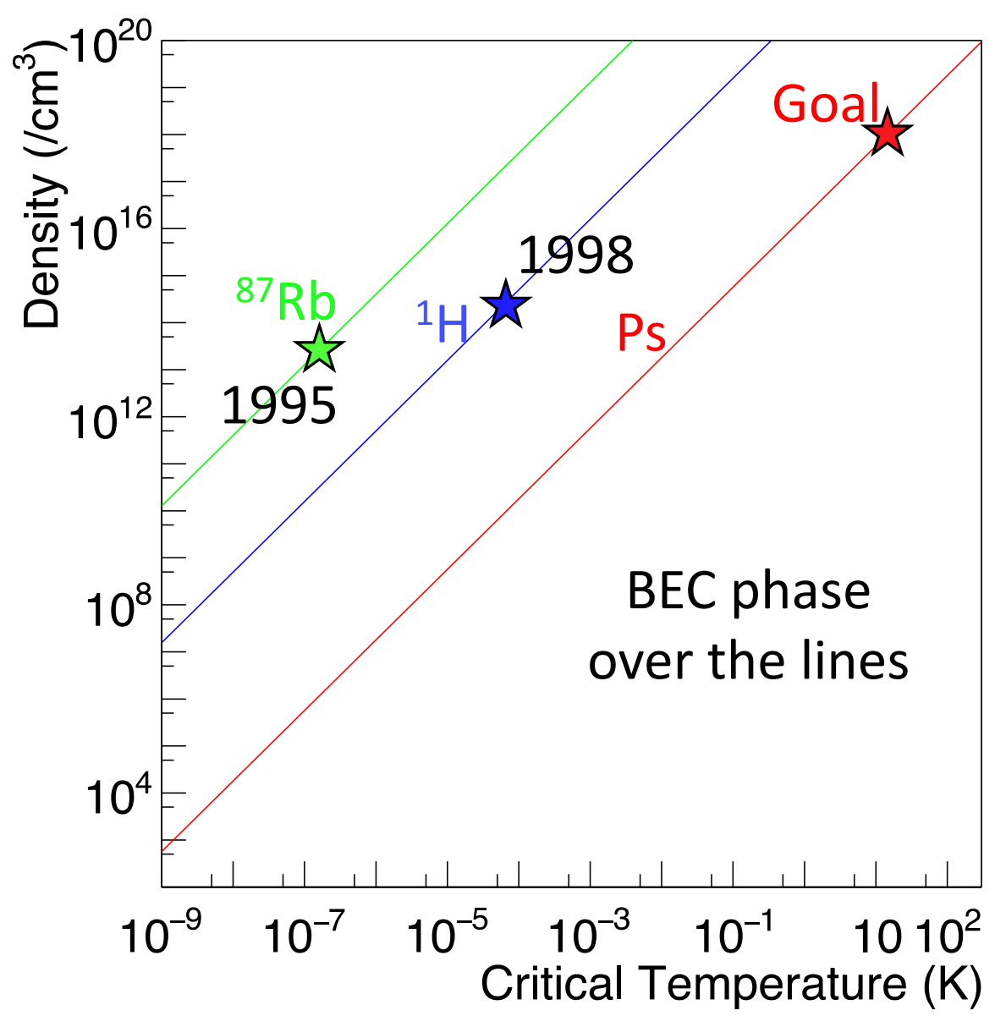


Fig. 1 BEC critical temperatures.

Monte Carlo simulation

Setup

- 5 keV, 10^7 e$^+$/bunch beam.
- Silica cavity (1 K) and cooling lasers.
- (100 nm)^3 cube internal void traps 4000 fully polarized Ps (n = 4 x 10^{18} cm^{-3}).

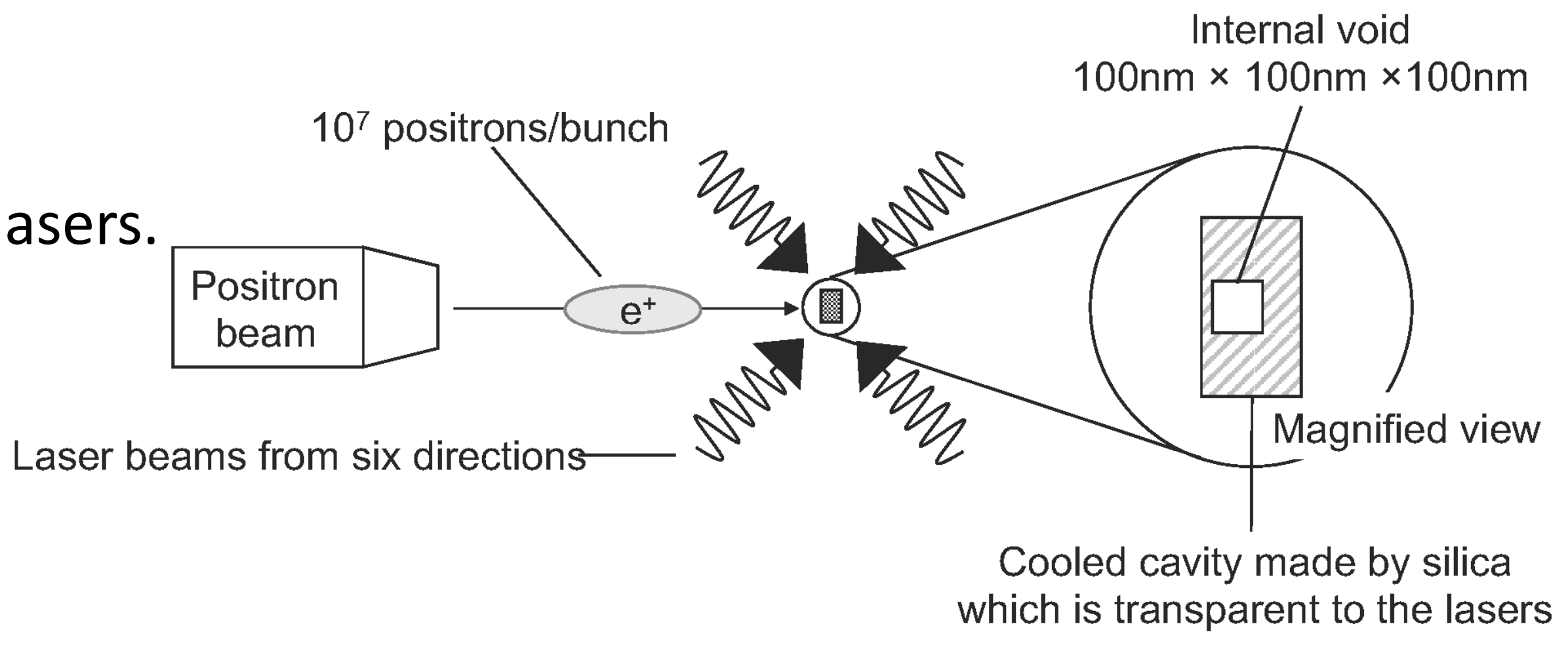


Fig. 2 Conceptual view of the setup.

Initial conditions

- 10^4 fully polarized Ps.
- 0.8 eV monochromatic kinetic energy.
- Isotropic velocities, uniform position distributions.

Method

- Velocity and internal state of every Ps are traced at the same time (brute-force method).
- Time evolutions are calculated with short time steps.
- Random numbers are generated at each step to determine whether and which interaction each Ps does.
- Step size is only ~0.1 ps, which is short so that probabilities of all interactions are less than 1%.
- One execution without averaging.
- Positions of Ps are not traced. 100 nm cube is much smaller than the laser beam size.
- Initial Ps number of 10^4 is larger than 4 x 10^3, which is the assumed number of fully polarized Ps survived in the silica cavity, in order to decrease the statistical uncertainty. Ps-Ps scattering rate is scaled to compensate the difference.

Interactions

1. Thermalization

- Ps lose energy by collisions with silica cavity walls.
- Each Ps kinetic energy is assumed to evolve by the same way as the mean kinetic energy.
- The following differential equation (2) is solved by the Euler method.

$$\frac{dE}{dt} = -\frac{2}{LM}\sqrt{2m_{Ps}E}\left(E - \frac{3}{2}k_B T\right), \quad (2)$$

E : Ps mean kinetic energy,
t : time since Ps formation,
$L = \sqrt[3]{V}$ (Ps mean free path),
V : volume of silica cavity,
M : effective mass of silica grains,
m_{Ps} : Ps mass,
T : sillica temperature.

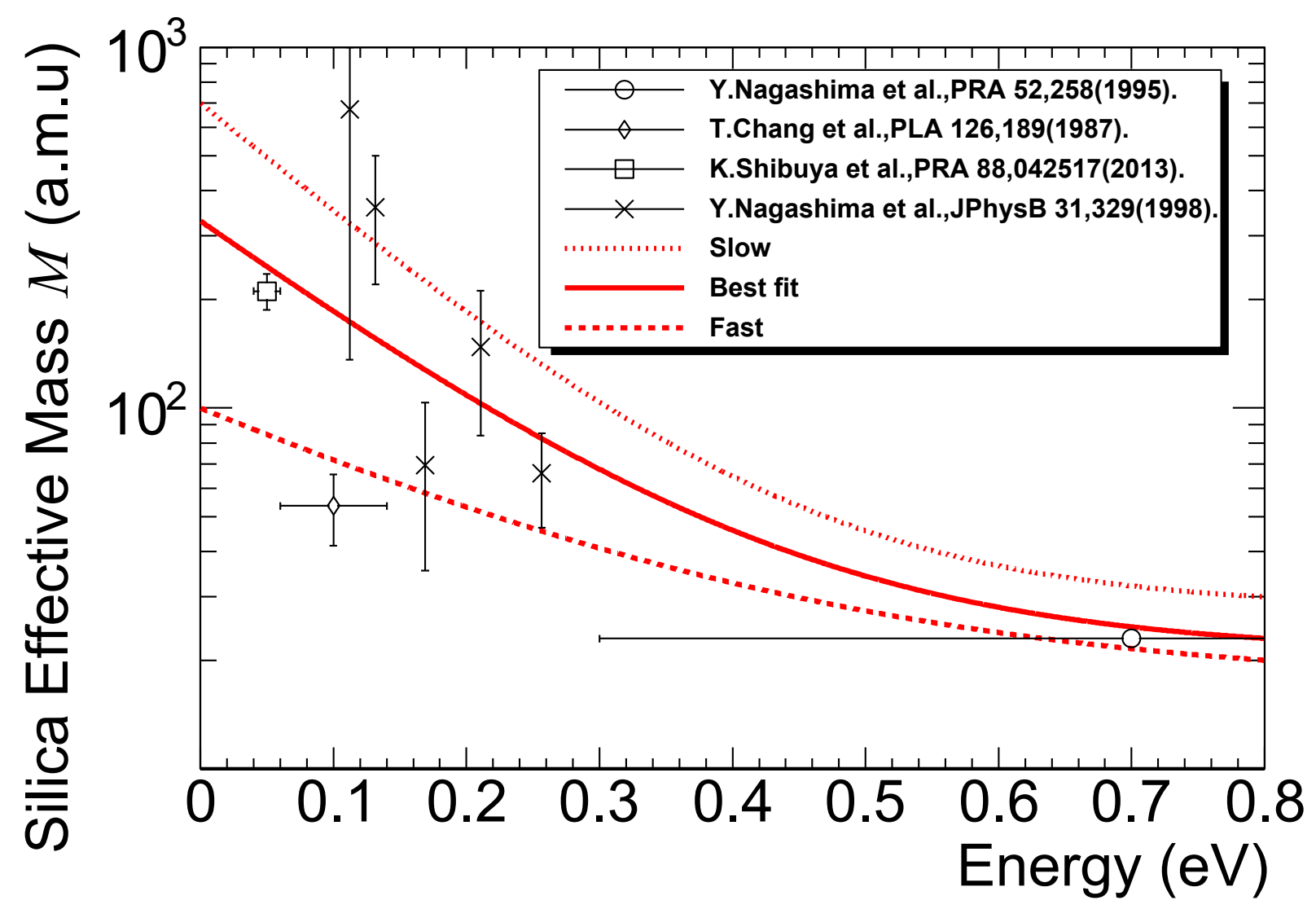


Fig. 3 Silica effective mass

- *M* is determined so that it represents the thermalization measurements (Fig. 3). The best-fit function is the following Eq. (3).

$$\frac{M}{\text{a.m.u.}} = 21 + 308\exp\left(-\frac{E}{0.16\ \text{eV}}\right) \quad (3)$$

2. Ps-Ps scatterings

- Two-body elastic *s*-wave scatterings.
- Cross section and mean free time are calculated by Eqs. (4) and (5).

$$\sigma = 4\pi a^2, \quad (4)$$

σ : scattering total cross section,
$a = 0.16$ nm (scattering length).

$$\tau = \frac{1}{n\sigma\bar{v}}, \quad (5)$$

τ : scattering mean free time,
$\bar{v}$: Ps mean velocity.

3. Ps decays

- 1*s* o-Ps annihilation (lifetime is 142 ns). Annihilated Ps are removed.
- 2*p* o-Ps annihilation is ignored because of long lifetime.
- Lyman- spontaneous de-excitation (3.2 ns) is included in the simulation.

4. Interaction with lasers

- Rates are calculated using Eq. (6) (right top of this poster).
- Laser intensity profile is approximated to be uniform at the peak intensity.

$$B\left(t,\vec{x},\vec{v}\right) = \int d\omega \frac{I\left(t,\vec{x},\omega\right)}{\hbar\omega}\cdot\frac{4}{3}\pi^2\alpha\omega_0\left|X_{12}\right|\cdot\frac{1}{2\pi} \times \frac{\Gamma/2}{\left[\omega\left(1-\hat{\vec{k}}\cdot\vec{v}/c\right)-\omega_0\right]^2+\left(\Gamma/2\right)^2}, \quad (6)$$

$B\left(t,\vec{x},\vec{v}\right)$: Einstein B coefficient,
$\vec{x}$: Ps position,
$\vec{v}$: Ps velocity,
ω : laser frequency,
$I\left(t,\vec{x},\omega\right)$: laser intensity per frequency,
α : fine structure constant,
ω_0 : $1s-2p$ resonant frequency,
X_{12} : $1s-2p$ transition matrix element,
$\Gamma = 313$ MHz (natural linewidth),
$\hat{\vec{k}}$: laser photons' direction,
c : speed of light.

Table 1 Assumed cooling laser specifications.

Parameters	Values
Pulse energy	40 μJ
Center frequency	1.23 PHz – Δ(*t*)
Frequency detune Δ(*t*)	Δ(0 ns) = 300 GHz Δ(300 ns) = 240 GHz
Bandwidth (2σ)	140 GHz
Time duration (2σ)	300 ns
Beam size (2σ)	200 μm

Results

Thermalization

- Thermalization is too slow.
- Huge uncertainty because of uncertainty of M.
- Precise measurement of Ps thermalization in silica cavity is necessary.

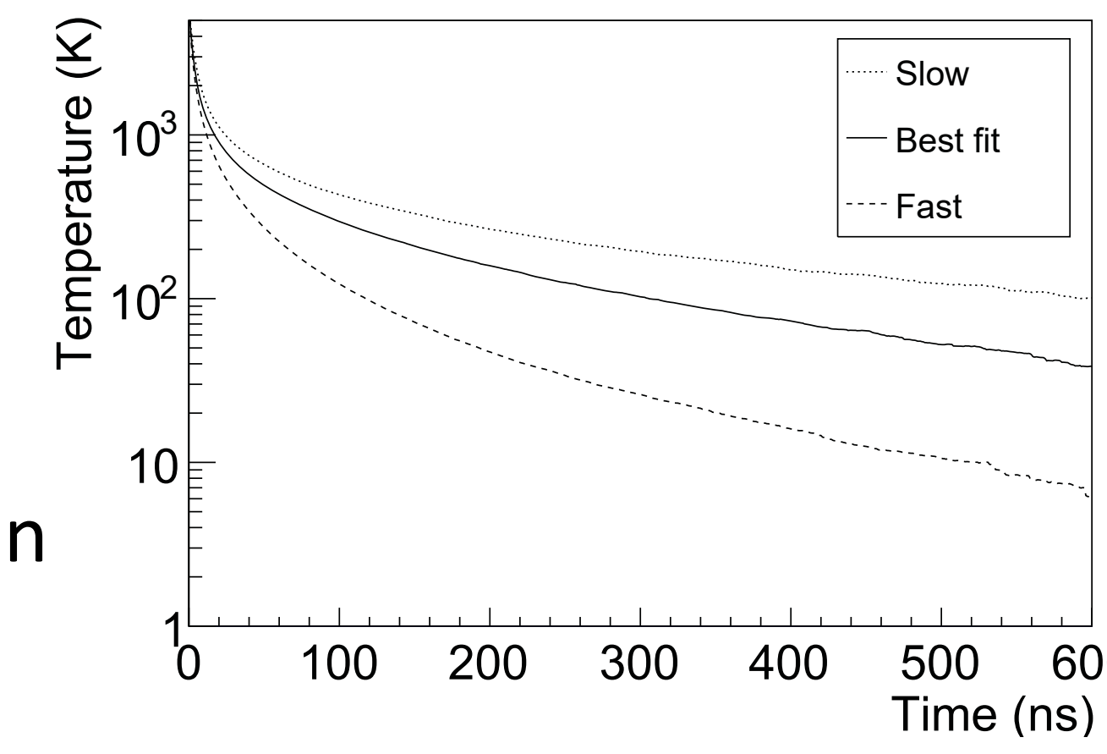


Fig. 4 Time evolution of Ps temperature (thermalization only).

Combination of Thermalization and Laser cooling

BEC can be realized by our new method (thermalization + laser cooling)!

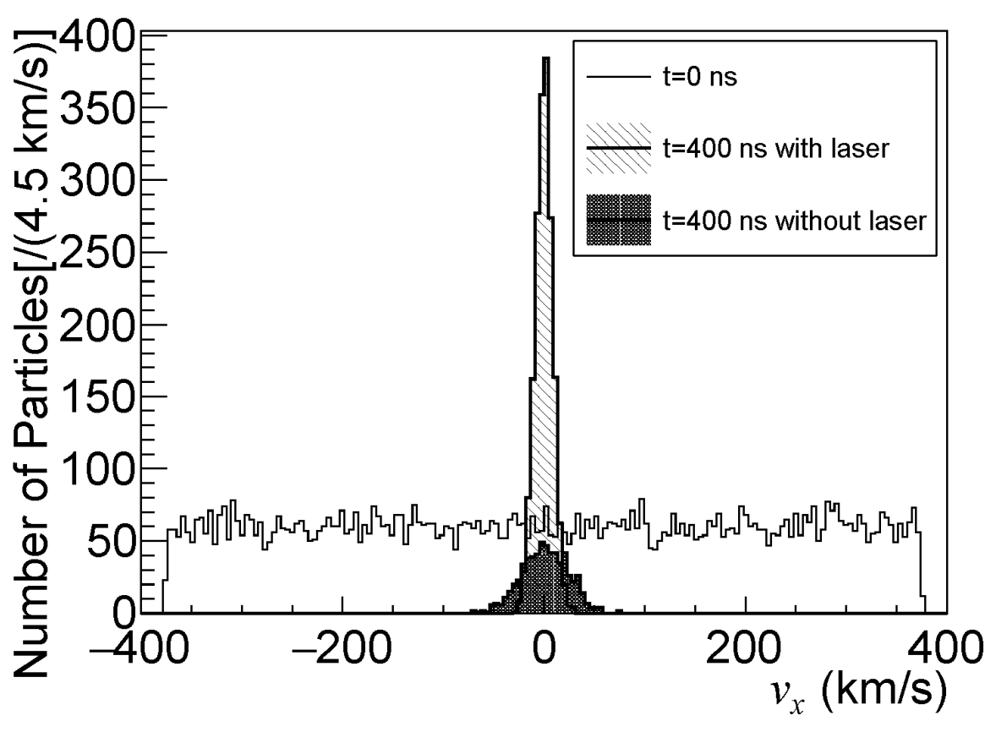


Fig. 5 Ps velocity distribution at several timings. Temperature is derived from the widths.

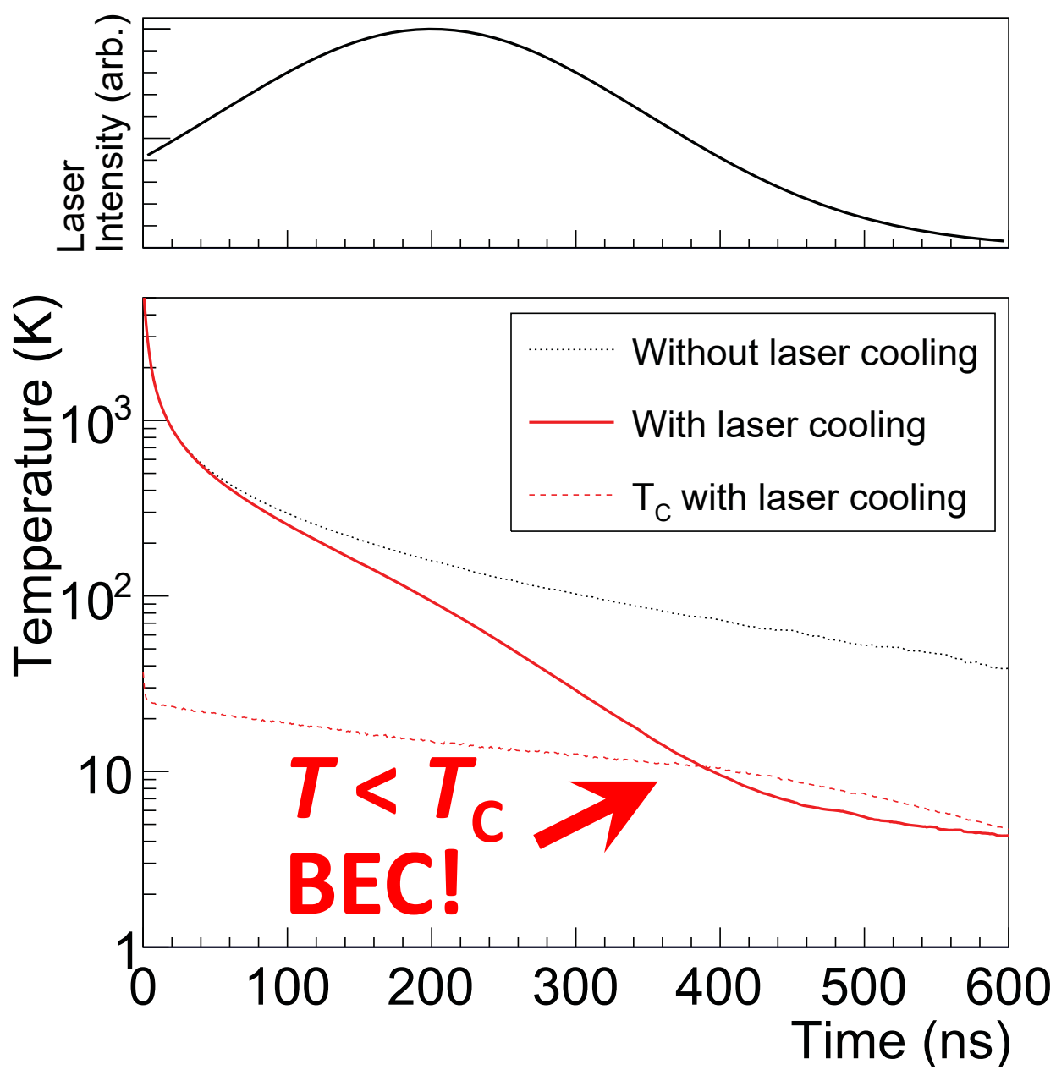


Fig. 6 Time evolution of laser intensity (top) and Ps temperature (bottom) with laser cooling (Best-fit *M* is used).

$$R_C = 1 - \left(T/T_C\right)^{\frac{3}{2}},$$

R_C : condensate fractions over remaining atoms.

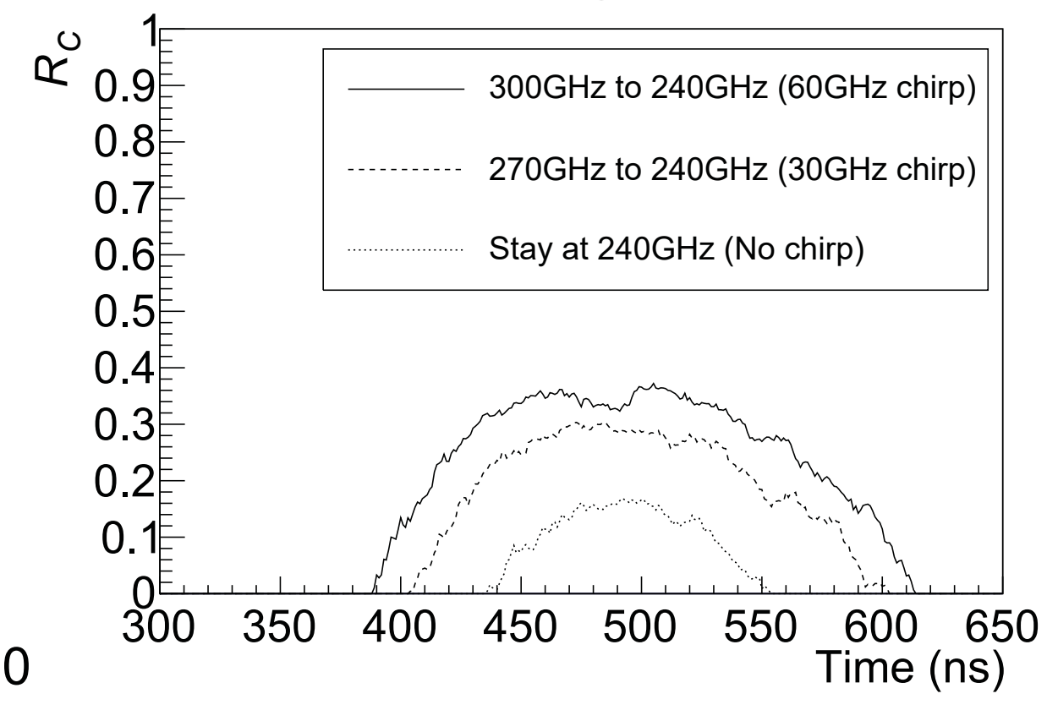
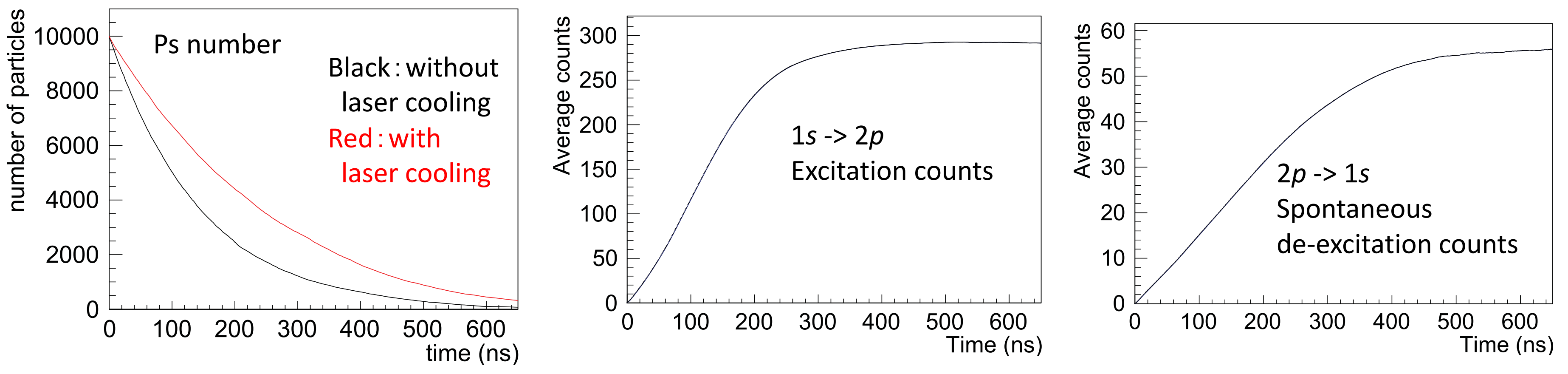


Fig. 7 Time evolution of BEC fractions with different laser frequency chirp conditions.

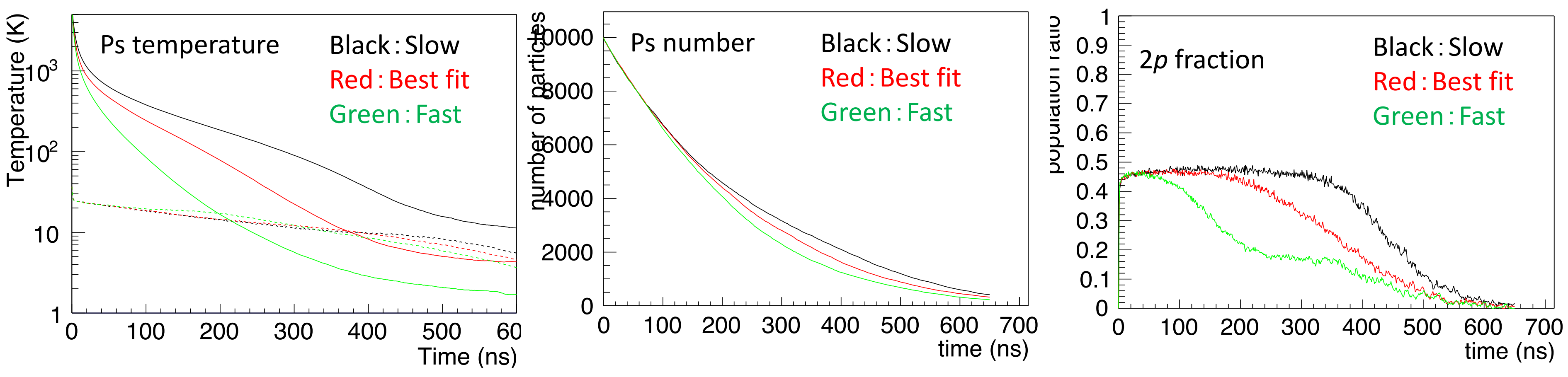
Frequency chirp is important!

Time evolutions of some parameters:

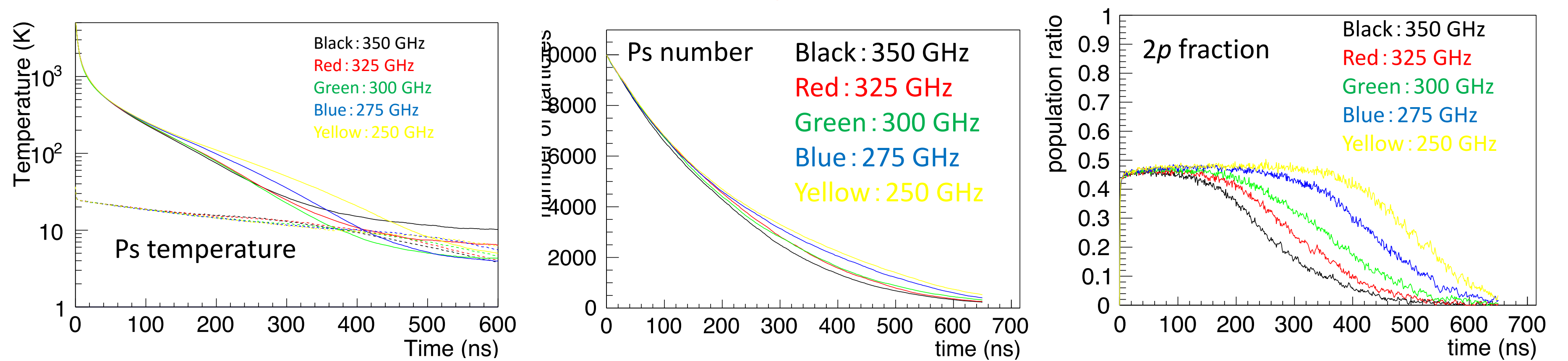


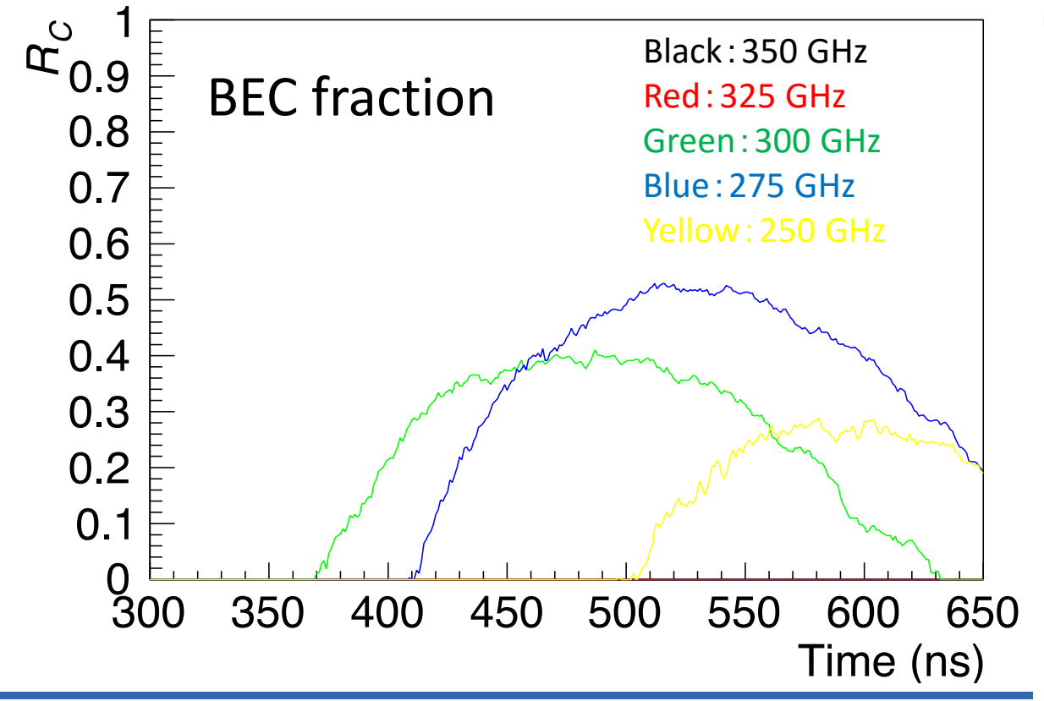
- Ps lifetime is longer with laser cooling because of 1*s*-2*p* transitions (2p has long lifetime).

Effect of uncertainty of silica effective mass *M*



Dependence on initial laser frequency





Reference:
K. Shu *et al.*, “Study on cooling of positronium for Bose-Einstein condensation”, J. Phys. B: At. Mol. Opt. Phys. **49**, 104001 (2016).

Summary

- We have studied a Ps cooling for BEC by a Monte Carlo simulation which includes thermalization, Ps-Ps scatterings, Ps decays and interaction with cooling lasers.
- Our simulation has demonstrated that **Ps BEC can be realized** by a **combination of the following 2 processes**, which is **our new idea** of Ps cooling:
  (1) **Ps thermalization** in small cold silica cavity
  (2) **Laser cooling** using 1*s*-2*p* transitions.